



# The thermal behavior of thin metal films in the hyperbolic two-step model

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Received 9 October 1998; received in revised form 6 April 1999

## Abstract

A new approach, based on the physical decoupling of the hyperbolic two-step model, is introduced to describe the thermal behavior of a thin metal film exposed to picoseconds thermal pulses. The approach is based on the assumption that the metal film thermal behavior occurs in two separate stages. In the first stage, electron gas transmits its energy to the solid lattice through electron–phonon coupling and other mechanisms of energy transport are negligible. In the second stage, electron gas and solid lattice are in thermal equilibrium, the energy transfer through electron–phonon coupling is negligible, and thermal diffusion dominates. The proposed approach eliminates the coupling between the energy equations and the reduced differential equations are easier to handle. The proposed approach applies to metal films whenever the dimensionless parameter  $GL^2/K_c$  is much larger than one. © 2000 Elsevier Science Ltd. All rights reserved.

## 1. Introduction

Energy transport during fast laser heating of solids has become an active research area beginning with the employment of short-pulse lasers in the fabrication of microstructures, laser patterning, laser processing of diamond films from carbon ion implanted copper substrates and laser surface hardening [1].

In applications involving high-rate heating induced by a short-pulse laser, the typical response time is an order of picoseconds [2,3] which is comparable to the phonon–electron thermal relaxation time. In such cases, thermal equilibrium between phonons and electrons cannot be assumed and heat transfer in the elec-

tron gas and the metal lattice needs to be considered separately. Models describing the non-equilibrium thermal behavior in such cases are called the microscopic two-step models. In the literature, there are two microscopic two-step models. The first one is the parabolic two-step model which is pioneered by Anisimov et al. [4] and advanced later by Fujimoto et al. [5]. The second one is the hyperbolic two-step model introduced by Qiu and Tien [3] based on the macroscopic averages of the electric and heat currents carried by electrons in the momentum space.

Laser heating of metals consists of two major steps of energy transfer which occur simultaneously. In the first step electrons absorb most of the incident radiation energy and the excited electron gas transmits its energy to the lattice through inelastic electron–phonon scattering process. In the second step, the incident radiation absorbed by the metal film diffuses spatially within the film mainly by the

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### Nomenclature

$C$	heat capacity, J/m <sup>3</sup> K
$\bar{C}$	propagation speed, m/s
$g$	electron–phonon coupling factor, W/m <sup>3</sup> K
$G$	Green’s function
$h$	Planck constant, J s
$K_B$	Boltzmann constant, J/K
$K$	thermal conductivity, W/m K
$L$	film thickness, m
$n_e$	electron number density per unit volume, 1/m <sup>3</sup>
$n_a$	atomic number density per unit volume, 1/m <sup>3</sup>
$q$	conduction heat flux, W/m <sup>2</sup>
$Q$	heat source, W/m <sup>3</sup>
$Q_0$	heat source amplitude, J/m <sup>3</sup>
$t$	time, s
$t_0$	first stage duration, s
$t^*$	time at which the energy of the laser beam is released, s
$T$	temperature, K
$T_D$	Debye temperature, K
$T_F$	Fermi temperature, K
$T_i$	initial temperature of both lattice and electron gas, K

$T_0$	electron and phonon temperature at the end of the first stage, K
$v_s$	speed of sound, m/s
$x$	spatial coordinate, m
$X$	dimensionless spatial coordinate, $\bar{C}_x/2\alpha$

### Greek symbols

$\alpha$	thermal diffusivity, m <sup>2</sup> /s
$\delta$	Dirac’s delta function
$\Delta T(x)$	difference in temperature, K
$\theta$	dimensionless temperature $(T - T_0(x))/Q_0/(C_e + C_l)$
$\nu_D$	Debye frequency, 1/s
$\tau$	dimensionless time, $\bar{C}^2(t - t_0)/2\alpha$
$\bar{\tau}$	relaxation time, s
$\bar{\tau}_F$	relaxation time evaluated at Fermi surface, s

### Subscripts

a	atom
e	electron
i	initial
l	lattice
0	the end of the first stage

electron gas. For typical metals, depending on the degree of electron–phonon coupling, it takes about 0.1 to 1 ps for electrons and lattice to reach thermal equilibrium. When the laser pulse duration is comparable with or less than this thermalization time, electrons and lattice are not in thermal equilibrium. Under these situations, both the parabolic and hyperbolic two-step models need to be considered simultaneously.

It is difficult to solve the coupled energy equations in the microscopic hyperbolic two-step model even after eliminating the coupling. The elimination of the coupling yields a single equation containing higher-order mixed derivatives in both time and space leading to complications in the solution procedures. However, in situations involving high electron–phonon coupling and low electron thermal conductivity, a simplification of the generalized governing equations of the hyperbolic two-step model is possible. This can be done by realizing that the thermal behaviour occurs in two separate steps. In the first stage, which is very short, the electron gas absorbs the incident radiation and transmits its energy to the metal lattice. The incident radiation raises the electron gas temperature, which in turn, causes a sharp drop in its thermal conductivity as reported for the case of most metals [6]. This implies

that the energy diffused by the electron gas may be neglected compared to that transmitted to the solid lattice during the first stage. As a result, the first stage is described by the hyperbolic two-step model excluding the diffusion term. The coupling between the energy equations of the hyperbolic two-step model is easily eliminated after dropping this diffusion. The reduced energy equations may then be used to obtain the non-equilibrium period, that is, the time required by both the electron and the lattice to reach the local thermal equilibrium. At the end of the first stage, both electron gas and solid lattice assume approximately the same temperature. The second stage begins following the nonequilibrium period. In this stage, the absorbed energy starts to diffuse within the thin film. Since both electron gas and solid lattice have the same temperature, the thermal behaviour of the thin film is now described by the hyperbolic macroscopic one-step model which consists of one energy equation containing the diffusion term. The initial condition for this model is obtained from the spatial electron or lattice temperature distribution given at the end of the non-equilibrium period.

The assumption that the mechanism of heat transfer in the first stage is controlled mainly by energy transfer from the electron gas to metal lattice and without any

thermal diffusion by the electron gas is justified by the following reasons: systems of high electron–phonon coupling factor have very small nonequilibrium periods. During this short period, the electrons have no opportunity to diffuse energy within the metal film. The diffusion process requires a relatively much longer period, and controls the thermal behaviour of the second stage.

The objective of the present work is to introduce a simplified approach which describes the thermal behaviour of a thin film exposed to picosecond thermal pulses. It is assumed that the thermal behaviour of these metal films occurs in two successive stages. In the first stage, the electron gas transmits its energy to the solid lattice, and at the end of this stage, both electron gas and solid lattice reach the state of thermal equilibrium. In the second stage, the energy transfer through electron–phonon coupling is negligible and the thermal diffusion becomes important. The proposed approach eliminates the coupling between the energy equations of both solid lattice and electron gas, and it is much easier to handle the reduced partial differential equations.

## 2. Analysis

Consider applications involving ultrafast laser heating of metal films. When the laser pulse duration is much shorter than the electron–phonon thermal relaxation time, the hot electrons are not in local thermal equilibrium with the lattice. Consequently, one describes the thermal behavior of a metal in terms of one energy equations for electrons, one energy equation for phonons and a constitutive law which relates the conduction heat flux with the temperature gradient in the electron gas. The coupling between the electron gas and solid lattice occurs via the electron–phonon interaction. The time evolution of the energies is given by the hyperbolic two-step model, which is proposed by Qiu and Tien [3], as follows:

$$C_i(T_i) \frac{\partial T_i}{\partial t} = g(T_e - T_i) \tag{1}$$

$$C_e(T_e) \frac{\partial T_e}{\partial t} = -\nabla q - g(T_e - T_i) + Q \tag{2}$$

$$\bar{\tau}_F \frac{\partial q}{\partial t} + K_e \nabla T_e + q = 0 \tag{3}$$

where  $\bar{\tau}_F$  is the relaxation time evaluated at Fermi surface [1,7] and  $g$  is the coupling factor which characterizes the energy exchange between phonons and electrons and is given as [1]:

$$g = \frac{\pi^4 (n_e v_s k_B)^2}{K_e}, \quad v_s = \frac{k_B}{2\pi h} (6\pi^2 n_a)^{-1/3} T_D \tag{4}$$

where  $n_e$  denotes the electron number density,  $v_s$  the speed of sound,  $k_B$  the Boltzmann constant,  $K_e$  the electron thermal conductivity,  $n_a$  the phonon density,  $h$  the Planck constant and  $T_D$  the Debye temperature. The coupling factor relative to electron diffusion is dimensionless and explains

$$\begin{aligned} \frac{gL^2}{K_e} &\sim \frac{g(T_e - T_i)}{K_e \frac{(T_e - T_i)}{L^2}} \\ &= \frac{\text{electron–phonon energy exchange}}{\text{net diffusion of electrons}} \end{aligned} \tag{5}$$

Note also that, as dimensional groups, the upper limit of sound propagation in a quantized solid is characterized by velocity

$$v_s \sim v_D n_a^{-1/3} \tag{6}$$

where  $v_D$  being the Debye frequency. This velocity can be rearranged in terms of the quantized energy, corresponding to the Debye temperature,

$$h v_D = k_B T_D, \tag{7}$$

to give

$$v_s \sim T_D \frac{k_B}{h} n_a^{-1/3} \tag{8}$$

which provides a dimensional interpretation of the sound velocity given in Eq. (4). Further note that, Eq. (5) rearranged in terms of Eq. (4) gives

$$\frac{gL^2}{K_e} \sim \pi^4 \left( \frac{n_e v_s k_B L}{K_e} \right)^2, \tag{9}$$

where, for a given temperature, the ratio

$$\frac{n_e v_s k_B T}{K_e T} \sim \frac{\text{electron energy flow}}{\text{electron diffusion}} = Pe \tag{10}$$

denotes a Peclet number which is a well-known dimensionless number in the phenomenological thermal science literature. In terms of this number, Eq. (9) may be interpreted as

$$\left( \frac{gL^2}{K_e} \right)^{1/2} \sim Pe \tag{11}$$

Elimination of the coupling among Eqs. (1)–(3) yields an equation with mixed derivatives. The high-order terms and mixed derivatives resulting from this elimination lead to solution difficulties. However, in some cases, the coupling between the two energy equations

may be eliminated without these difficulties. These cases involve very large coupling and very short pulse duration.

A simplified solution procedure is presented here for elimination of the coupling amount Eqs. (1)–(3). This is done by recognizing the fact that the thermal behavior of the thin metal film occurs in two separate stages. In the first stage, which has very short duration, electron gas absorbs the incident laser radiation and transmits its energy to the solid lattice. The incident radiation raises the temperatures of the electron gas, which in turn causes a drop in its thermal conductivity  $K_e$ . As an example, it is reported [6] that the electron gas thermal conductivity for highly pure copper drops from  $2 \times 10^4$  W/m K at  $T = 10$  K to  $5 \times 10^2$  W/m K at  $T = 1000$  K. Also, a very short thermal pulse leads to a very short period for the first stage. During this stage, energy diffusion in electron gas and solid lattice is negligible. Eqs. (1) and (2) are then reduced to

$$C_1(T_1) \frac{\partial T_1}{\partial t} = g(T_e - T_1) \quad (12)$$

$$C_e(T_e) \frac{\partial T_e}{\partial t} = -g(T_e - T_1) + Q \quad (13)$$

which are valid for  $0 \leq t \leq t_0$ , where  $t_0$  is the period of the first stage. Note that the diffusion becomes negligible whenever the ratio of exchanged energy through electron–phonon coupling to diffused energy through conduction in electron gas is much larger than 1, that is,

$$\frac{gL^2}{K_e} \gg 1$$

Table 1 shows this criterion, at room temperature, for different metal films of thickness  $1 \times 10^{-6}$  m [1].

To describe the thermal behavior during the first stage, combine Eqs. (12) and (13) to yield

$$C_1(T_1) \frac{\partial T_1}{\partial t} + C_e(T_e) \frac{\partial T_e}{\partial t} = Q \quad (14)$$

subject to the initial conditions

$$T_e(0, x) = T_1(0, x) = T_i$$

At the end of the first stage  $t_0$ , it is assumed that the difference between the electron gas temperature ( $T_e(t_0, x)$ ) and solid lattice temperature  $T_1(t_0, x)$  is a small difference  $\Delta T(x)$ , where

$$\Delta T(x) = T_e(t_0, x) - T_1(t_0, x)$$

with  $(\Delta T/T_e) \ll 1$  and  $(\Delta/T_1) \ll 1$ . Integration of Eq. (14) over the first stage period yields

$$\int_{T_i}^{T_e(t_0, x) - \Delta T(x)} C_1 dT_1 + \int_{T_i}^{T_e(t_0, x)} C_e dT_e = \int_0^{t_0} Q dt \quad (15)$$

A substitute for  $T_1$  from Eq. (13) into (14), yields

$$\frac{\partial}{\partial t} \left( \frac{C_e}{g} \frac{\partial T_e}{\partial t} \right) + \left( 1 + \frac{C_e}{C_1} \right) \frac{\partial T_e}{\partial t} = \frac{\partial}{\partial t} \left( \frac{Q}{g} \right) + \frac{Q}{C_1} \quad (16)$$

Eqs. (15) and (16) need to be solved at each spatial location  $x$  to obtain the first stage period  $t_0$  and the temperature distribution  $T(t_0, x) = T_0 = T_e(t_0, x)$  at the end of the first stage. Note that each spatial location has different  $t_0$ , and one has to select the largest  $t_0$  in order to secure the state of thermal equilibrium between the electron gas and the solid lattice at each spatial location of the film. It is assumed that second stage will be activated when the temperature difference between the lattice and the electron is very small, but not zero. Waiting for the lattice to attain the exact temperature of the electron gas, that is  $\Delta T = 0$ , increases the duration of the first stage period because the energy exchange between electron and lattice becomes very slow at the end of the first stage due to the decrease in the temperature difference between them. As a result, thermal diffusion within the film, which is assumed to be absent during the first stage, may be activated and this violates the basic assumption that controls the thermal behavior of the first stage.

For constant thermal properties  $C_e$ ,  $C_1$  and  $g$ , Eq. (16) is solved as

Table 1

	Pb	Ag	Cu	Au
$K_e$ (W m <sup>-1</sup> K <sup>-1</sup> )	35	419	386	315
$g$ (W m <sup>-3</sup> K <sup>-1</sup> )	$12.4 \times 10^{16}$	$2.8 \times 10^{16}$	$4.8 \times 10^{16}$	$2.8 \times 10^{16}$
$gL^2/K_e$	3542	66	124	88

$$T_e(t, x) = c_1 + c_2 e^{-\Lambda t} + \int \frac{1}{\Lambda} \left( \frac{\dot{Q}}{C_e} + \frac{Qg}{C_1 C_e} \right) dt - e^{-\Lambda t} \int \frac{1}{\Lambda} \left( \frac{\dot{Q}}{C_e} + \frac{Qg}{C_1 C_e} \right) e^{\Lambda t} dt \tag{17}$$

where  $c_1$  and  $c_2$  are obtained using the following initial conditions

$$T_e(0, x) = T_i, \quad \frac{\partial T_e}{\partial t}(0, x) = 0$$

and  $\Lambda = (g/C_e)[1 + (C_e/C_1)]$ . Expression for  $T_1(t, x)$  is obtained directly from Eq. (13), with

$$\frac{\partial T_e}{\partial t}(t, x) = -\Lambda c_2 e^{-\Lambda t} + e^{-\Lambda t} \int \left( \frac{\dot{Q}}{C_e} + \frac{Qg}{C_1 C_e} \right) e^{\Lambda t} dt$$

Also, Eq. (15) is integrated to yield

$$T_e(t_0, x) = T_0(x) = T_i + \frac{\int Q dt}{C_e + C_1} + \frac{C_1}{C_1 + C_e} \Delta T(x) \tag{18}$$

It is worth mentioning here that the drop in the temperature of the electron gas, which occurs at the end of the first stage, causes an increase in the electron gas thermal conductivity. As a result, the thermal behavior of the second stage is activated. The governing equation which describes the thermal behavior of the thin film during the second stage is the classical hyperbolic energy equation replacing Eqs. (1), (2) and (3),

$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial q}{\partial x} \tag{19}$$

$$\bar{\tau} \frac{\partial q}{\partial t} + K \frac{\partial q}{\partial x} + q = 0 \tag{20}$$

Note that the heating source term does not appear in Eq. (19) because the duration of the incident laser radiation is less than  $t_0$ . Eqs. (19) and (20) are combined to yield in one dimensional form,

$$\frac{1}{\bar{C}^2} \frac{\partial^2 T}{\partial t^2} + \frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \tag{21}$$

where  $\bar{C}^2 = (\alpha/\bar{\tau})$ . The initial conditions for Eq. (21) are obtained from the temperature distribution  $T(t_0, x) = T_0(x)$  and its derivative  $[\partial T(t_0, x)/\partial t] = [\partial T_0(x)/\partial t]$  at the end of the first stage. However, at the end of the first stage,  $T_e(t_0, x) \approx T_e(t_0, x)$  and  $Q = 0$ , and as a result of Eqs. (12) and (13),

$$\frac{\partial T(t_0, x)}{\partial t} = \frac{\partial T_0(x)}{\partial t} = \frac{\partial T_e(t_0, x)}{\partial t} = \frac{\partial T_1(t_0, x)}{\partial t} = 0$$

The assumption that the heating source vanishes at the

end of the first stage is an important condition since it is impossible for  $T_1$  to reach  $T_e$  during the evolving of the heating source.

We now proceed to an example illustrating the simple approach developed in this study.

### 3. Illustrative example

Consider a very short laser pulse on a pure metal film of thickness  $L$  having a high coupling factor  $g$ . The metal film is analyzed on the basis of a one-dimensional model since, in a very short time, the beam diameter is typically much larger than the penetration depth of diffusion. Neglecting the temperature dependence of thermal properties, assuming the incident radiation to be totally absorbed by the electron gas and neglecting thermal diffusion within the solid lattice, the governing equations are Eqs. (1)–(3) with constant thermal properties  $c_e$ ,  $C_1$ ,  $g$  and  $K_e$ . The source term that appears in these equations may be approximated as

$$Q(t, x) = Q_0(x)\delta(t - t^*) \tag{22}$$

where  $\delta(t-t^*)$  is the Dirac's delta function which assumes that the incident laser beam evolves all of its energy at time  $t^*$  and  $Q_0(x)$  represents the spatial variation in the heating source which is rewritten as

$$Q_0(x) = Q_0 f(x) \tag{23}$$

where  $f(x)$  represents any spatial variation in the heating source term. Let  $f(x)$  be expanded in terms of Fourier cosine half range expansion, as

$$f(x) = a_0 + \sum_{n=1}^{n=\infty} a_n \cos \frac{n\pi x}{L} \tag{24}$$

where

$$a_0 = \frac{1}{L} \int_0^L f(x) dx, \quad a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx$$

Insert the specified heating source, given by Eq. (22), into Eqs. (17) and (18), and carry out the required integrations, which yields

$$T_e(t_0, x) = T_0(x) = T_i + Q_0 f(x) [B_1 + B_2 e^{-\Lambda(t-t^*)}], \quad \text{for } t > t^*$$

$$T_e(t_0, x) = T_0(x) = 0, \quad \text{for } t < t^* \tag{25}$$

where

$$B_1 = \frac{1}{C_1 + C_e}, \quad B_2 = \frac{C_1}{C_e(C_1 + C_e)}$$

$$T_e(t_0, x) = T_0(x) = T_1 + \frac{Q_0 f}{C_1 + C_e} + \frac{C_1}{C_1 + C_e} \Delta T(x) \quad (26)$$

The duration of the first stage  $t_0$  is found by equating Eq. (25) to Eq. (26) and solving for  $t = t_0$ , to yield

$$t_0 = t^* - \frac{1}{\Lambda} \ln \left[ \frac{C_e \Delta T(x)}{Q_0 f} \right] \quad (27)$$

During the first stage, thermal diffusion is absent, and as a result, one may assume that the temperature difference  $\Delta T(x)$  has the same spatial distribution as the spatial distribution of the heating source. This implies that

$$\Delta T(x) = f(x) \Delta T$$

and Eq. (27) is reduced to

$$t_0 = t^* - \frac{1}{\Lambda} \ln \left[ \frac{C_e \Delta T}{Q_0} \right]$$

Expression for  $T_1(t, x)$  is obtained directly from Eq. (13), with

$$\frac{\partial T_e}{\partial t}(t, x) = -Q_0 f(x) B_2 \Lambda e^{-\Lambda(t-t^*)}, \quad \text{for } t > t^*$$

$$\frac{\partial T_e}{\partial t}(t, x) = 0, \quad \text{for } t < t^*$$

Also, the governing equations describing the thermal behavior of the thin film during the second stage and are given by Eq. (21) with the following initial and boundary conditions

$$T(t_0, x) = T_0(x), \quad \frac{\partial T}{\partial t}(t_0, x) = 0$$

$$\frac{\partial T}{\partial x}(t, 0) = \frac{\partial T}{\partial x}(t, L) = 0 \quad (28)$$

Now, using the following dimensionless parameters,

$$\theta = \frac{T - T_0(x)}{C_e + C_1}, \quad \tau = \frac{\bar{C}^2(t - t_0)}{2\alpha}, \quad X = \frac{\bar{C}x}{2\alpha}$$

Eqs. (21) and (28) are reduced to

$$\frac{\partial^2 \theta}{\partial \tau^2} + 2 \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial X^2} + f''(X) \quad (29)$$

$$\theta(0, X) = 0, \quad \frac{\partial \theta}{\partial \tau}(0, X) = 0$$

$$\frac{\partial \theta}{\partial X}(\tau, 0) = \frac{\partial \theta}{\partial X}(\tau, X_1) = 0 \quad (30)$$

where  $f''(X) = -\sum_{n=1}^{\infty} a_n \cos \gamma_n X$ , and  $\gamma_n = 2n\pi\alpha/L\bar{C}$ . The boundary conditions in (30) are obtained as a result of the fact that  $f'(0) = f'(X_1) = 0$ . The analytical solution to Eqs. (29) and (30) is expressed in terms of Green's function (see, for example, Ozisik [8])

$$\theta(\tau, X) = \int_{X'=0}^1 \int_{\tau^*=0}^{\tau} G(X, \tau | X', \tau^*) f''(X') d\tau^* dX' \quad (31)$$

Here  $G(X, \tau | X', \tau^*)$  is the appropriate Green's function for the solution of governing Eq. (29) and its initial and boundary conditions with the source term replaced by the unit impulse function  $\delta(\tau - \tau^*)\delta(X - X')$ . In terms of the Green function, the film temperature given by Eq. (29) satisfies

$$\frac{\partial^2 G}{\partial \tau^2} + 2 \frac{\partial G}{\partial \tau} - \frac{\partial^2 G}{\partial X^2} = \delta(\tau - \tau^*)\delta(X - X') \quad (32)$$

Subject to the following initial and boundary conditions:

$$\frac{\partial G}{\partial X} = 0, \quad X = 0$$

$$\frac{\partial G}{\partial X} = 0, \quad X = X_1$$

$$G = 0, \quad \tau < \tau^*$$

$$\frac{\partial G}{\partial \tau} = 0, \quad \tau < \tau^* \quad (33)$$

The initial conditions are based on the causality principle, which states that there can be no effect experienced at times prior to the cause. The finite integral transforms are used to solve systems (32) and (33). Consider the integral transform and inversion pair defined as (see, for example, Ozisik [8]):

$$\bar{G}_m(\tau) = \int_{X'=0}^{X_1} G(X, \tau | X', \tau^*) \cos(\lambda_m X) dX \quad (34)$$

$$G(X, \tau | X', \tau^*) = \sum_{m=0}^{\infty} \frac{\bar{G}_m(\tau)}{N(\lambda_m)} \cos(\lambda_m X) \quad (35)$$

where  $N(\lambda_m)$  is the normalization integral given by

$$N(\lambda_m) = X_1 \quad \text{for } m = 0, \quad \text{and} \quad N(\lambda_m) = \frac{X_1}{2} \quad \text{for} \quad (36)$$

$$m = 1, 2, 3, \dots$$

and  $\lambda_m$ s are the eigenvalues by

$$\lambda_m = \frac{m\pi}{X_1} \tag{37}$$

The transform of Eq. (32) satisfies

$$\begin{aligned} \frac{d^2 \bar{G}_m(\tau)}{d\tau^2} + 2 \frac{d\bar{G}_m(\tau)}{d\tau} + \lambda_m^2 \bar{G}_m(\tau) \\ = \cos(\lambda_m X') \delta(\tau - \tau^*) \end{aligned} \tag{38}$$

subject to

$$\bar{G}_m(\tau) = 0, \quad \tau < \tau^*$$

$$\frac{d\bar{G}_m(\tau)}{d\tau} = 0, \quad \tau < \tau^* \tag{39}$$

By a somewhat lengthy but straightforward manipulation, the solution to the system of Eqs. (38) and (39) leads to

$$\begin{aligned} \bar{G}_m(\tau) = \frac{\cos(\lambda_m X')}{\beta_m} e^{-(\tau-\tau^*)} \sin(\beta_m(\tau - \tau^*)), \\ \tau > \tau^* \end{aligned} \tag{40}$$

where  $\beta_m = \sqrt{\lambda_m^2 - 1}$ . The inversion formula given by Eq. (35) readily gives the Green function as

$$\begin{aligned} G(X, \tau | X', \tau^*) = \frac{1}{X_1} \\ e^{-(\tau-\tau^*)} \sinh(\tau - \tau^*) + \sum_{m=1}^{\infty} 2 \frac{\cos(\lambda_m X')}{\beta_m X_1} \\ e^{-(\tau-\tau^*)} \sin(\beta_m(\tau - \tau^*)) \cos(\lambda_m X), \quad \tau > \tau^* \end{aligned} \tag{41}$$

In terms of this solution, Eq. (31) yields

$$\begin{aligned} \theta(\tau, X) = \frac{1}{X_1} \int_{X'=0}^1 \int_{\tau^*=0}^{\tau} \\ e^{-(\tau-\tau^*)} \sinh(\tau - \tau^*) f''(X') d\tau^* dX' \\ + \sum_{m=1}^{\infty} 2 \frac{\cos(\lambda_m X')}{\beta_m X_1} \\ e^{-(\tau-\tau^*)} \sin(\beta_m(\tau - \tau^*)) \cos(\lambda_m X) f''(X') d\tau^* dX' \end{aligned} \tag{42}$$

The first term on the RHS of Eq. (42) represents the steady-state part of the solution remaining after the transients have died out. Any energy released within any insulated region will merely distribute evenly over the entire region after sufficient time. The temperature distribution is directly available by inserting Eq. (24) into the general solution (42). After performing indicated operations

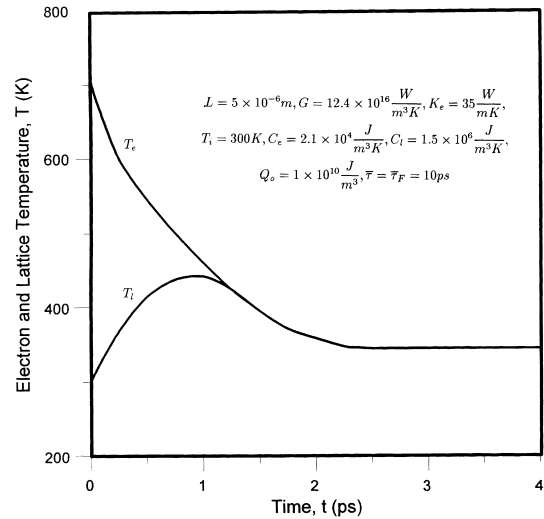


Fig. 1. Transient electron and lattice temperatures variation at the surface of an insulated thin lead film.

$$\begin{aligned} \theta(\tau, X) = -\frac{1}{X_1} \left[ -\frac{1}{4} + \frac{\tau}{2} + \frac{1}{4} e^{-2\tau} \right] \sum_{n=1}^{n=\infty} \frac{a_n}{\gamma_n} \sin \gamma_n X_1 \\ - \sum_{m=1}^{\infty} \frac{2}{X_1} \left[ \frac{1}{1 + \beta_m^2} - \frac{1}{\beta_m(1 + \beta_m^2)} e^{-\tau} (\sin \beta_m \tau + \beta_m \cos \beta_m \tau) \right] \\ \cos(\lambda_m X) \left\{ \sum_{n=1}^{\infty} \frac{a_n}{2} \left[ \frac{\sin(\lambda_m - \gamma_n) X_1}{\lambda_m - \gamma_n} + \frac{\sin(\lambda_m + \gamma_n) X_1}{\lambda_m + \gamma_n} \right] \right\} \end{aligned} \tag{43}$$

A sample of the results is presented in Fig. 1 for a thin metal film of lead exposed to a laser pulse, as in Eqs. (22) and (23), with  $f(x)=1$  for  $0 < x < (L/2)$  and  $f(x)=0$  for  $(L/2) < x < L$ . The results are obtained using the following values for different parameters:

$$L = 5 \times 10^{-6} \text{ m}, \quad g = 12.4 \times 10^{16} \text{ W/m}^3 \text{ K}, \quad K_e = 35 \text{ W/m K},$$

$$T_i = 300 \text{ K},$$

$$C_e = 2.1 \times 10^4 \text{ J/m}^3 \text{ K}, \quad C_l = 1.5 \times 10^6 \text{ J/m}^3 \text{ K},$$

$$Q_0 = 1 \times 10^{10} \text{ J/m}^3,$$

$$\bar{\tau} = \bar{\tau}_F = 10 \text{ ps}$$

#### 4. Concluding remarks

A simplified approach is introduced to describe the thermal behavior of a thin film exposed to a picosecond duration thermal pulse. It is assumed that the

film thermal behavior occurs in two separate stages. In the first stage, electron gas transmits its energy to the solid lattice, excluding any other mechanism of energy transport. In the second stage, the energy transfer through electron–phonon coupling is negligible and the energy transfer by diffusion becomes dominant. The dimensionless parameter which specifies the validity of the proposed model is found to be a sort of Peclet number which is equal to the square root of  $GL^2/K_e$ . This modified Peclet number describes the ratio of electron energy flow to electron energy diffusion. The proposed approach may be applied on metal films having modified Peclet number much larger than one. This is believed to be true for thin films having very high coupling factor, very short thermal pulse duration and relatively low or moderate thermal conductivity.

Using the simplified proposed model, the film thermal behavior is described by two ordinary differential equations (one for the electron gas and the other for the solid lattice) during the first stage and by one classical hyperbolic energy equation during the second stage.

Also, closed form expressions are derived for the duration of the first stage and the temperature distribution at the end of the first stage.

To demonstrate the validity of the proposed model, an illustrative example describes the thermal behavior of a metal film exposed to an impulsive laser heating source is given. The thermal behavior during the first stage is obtained directly from the general analysis and

that during the second stage is obtained using the Green's function method. The results reveal the existence of two separate stages where the temperature difference between electron and lattice vanishes at the beginning of the second stage.

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